ADJOINT QCD$_2$ IN LARGE N [1]

Stephen Pinsky

Department of Physics
The Ohio State University
174 West 18th Avenue
Columbus, Ohio 43210

Abstract

We consider a dimensional reduction of 3+1 dimensional $SU(N)$ Yang-Mills theory coupled to adjoint fermions to obtain a class of $1+1$ dimensional gauge theories. We derive the quantized light-cone Hamiltonian in the light-cone gauge $A_{-}=0$ and large-N limit, then solve for the masses, wavefunctions and of the color singlet boson and fermion boundstates. We find that the theory has many exact massless state that are similar to the t'Hooft pion.

INTRODUCTION

In this work [1], we start by considering QCD$_{3+1}$ coupled to Dirac adjoint fermions. Here, the virtual creation of fermion-antifermion pairs is not suppressed in the large-N limit – in contrast to the case for fermions in the fundamental representation [2] – and so one may study the structure of boundstates beyond the valence quark (or quenched) approximation[1]. We also anticipate that the techniques employed here will have special interest in the context of solving supersymmetric matrix theories.

The QCD$_{3+1}$ theory coupled to adjoint fermions is reduced to a $1+1$ dimensional field theory by stipulating that all fields are independent of the

[1]Talk presented at the 1997 Orbis Scientiae Conference Miami Beach Florida, January 25-28,1997; based on work done with F. Antonuccio

transverse coordinates $x^\perp = (x^1, x^2)$. The resulting theory is QCD_{1+1} coupled to two $1+1$ dimensional complex adjoint spinor fields, and two real adjoint scalars. A key strategy in formulating this model field theories is to retain as many of the essential degrees of freedom of higher dimensional QCD while still being able to extract complete non-perturbative solutions. One finds Yukawa interactions between the scalars and fermion fields. While this approach is not equivalent to solving the full $3+1$ theory and then going to the regime where $k_\perp$ is relatively small, it may share many qualitative features of the higher dimensional theory, since the longitudinal dynamics is treated exactly. Studies of this type for pure glue and with fundamental quarks have yielded a number of interesting results [2, 3, 4].

The unique features of light-front quantization [5] make it a powerful tool for the nonperturbative study of quantum field theories. The main advantage of this approach is the apparent simplicity of the vacuum state. Indeed, naive kinematic arguments suggest that the physical vacuum is trivial on the light front. Since in this case all fields transform in the adjoint representation of $SU(N)$, the gauge group of the theory is actually $SU(N)/Z_N$, which has nontrivial topology and vacuum structure. For the particular gauge group $SU(2)$ this has been discussed elsewhere [6]. While this vacuum structure may in fact be relevant for a discussion on condensates, for the purposes of this calculation they will be ignored.

In the first section we formulate the $3+1$ dimensional $SU(N)$ Yang-Mills theory and then perform dimensional reduction to obtain a $1+1$ dimensional matrix field theory. The light-cone Hamiltonian is then derived for the light-cone gauge $A_- = 0$ following a discussion of the physical degrees of freedom of the theory. Singularities from Coulomb interactions are regularized in a natural way, and we outline how particular "ladder-relations" take care of potentially troubling singularities for vanishing longitudinal momenta $k^+ = 0$. In the final section exact massless solutions of the boundstate integral equations are discussed.

DEFINITIONS

We first consider $3+1$ dimensional $SU(N)$ Yang-Mills coupled to a Dirac spinor field whose components transform in the adjoint representation of $SU(N)$:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}(\bar{\Psi}\gamma^{\mu} \overleftrightarrow{D}_{\mu} \Psi) - m\bar{\Psi}\Psi\right] \ , \tag{1}$$

where $D_{\mu} = \partial_{\mu} + \mathrm{i}g[A_{\mu}, \ \ ]$ and $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} + \mathrm{i}g[A_{\mu}, A_{\nu}]$. We also write $A_{\mu} = A_{\mu}^{a}\tau^{a}$ where τ^{a} is normalized such that $\mathrm{Tr}(\tau^{a}\tau^{b}) = \delta_{ab}$. The projection operators[2] Λ_L, Λ_R permit a decomposition of the spinor field $\Psi = \Psi_L + \Psi_R$, where

$$\Lambda_L = \frac{1}{2}\gamma^{+}\gamma^{-}, \quad \Lambda_R = \frac{1}{2}\gamma^{-}\gamma^{+} \quad \text{and} \quad \Psi_L = \Lambda_L\Psi, \quad \Psi_R = \Lambda_R\Psi. \tag{2}$$

Inverting the equation of motion for Ψ_L, we find

$$\Psi_L = \frac{1}{2\mathrm{i}D_{-}}\left[\mathrm{i}\gamma^{i}D_i + m\right]\gamma^{+}\Psi_R \tag{3}$$

where $i = 1, 2$ runs over transverse space. Therefore Ψ_L is not an independent degree of freedom.

Dimensional reduction of the $3+1$ dimensional Lagrangian (1) is performed by assuming (at the classical level) that all fields are independent of the transverse coordinates $x^{\perp} = (x^1, x^2)$: $\partial_{\perp}A_{\mu} = 0$ and $\partial_{\perp}\Psi = 0$. In the resulting $1+1$ dimensional field theory, the transverse components $A_{\perp} = (A_1, A_2)$ of the gluon field will be represented by the $N \times N$ complex matrix fields $\phi_{\pm}$:

$$\phi_{\pm} = \frac{A_1 \mp \mathrm{i}A_2}{\sqrt{2}}. \tag{4}$$

Here, ϕ_{-} is just the Hermitian conjugate of ϕ_{+}. When the theory is quantized, $\phi_{\pm}$ will correspond to ± 1 helicity bosons (respectively).

[2] We use the conventions $\gamma^{\pm} = (\gamma^0 \pm \gamma^3)/\sqrt{2}$, and $x^{\pm} = (x^0 \pm x^3)/\sqrt{2}$.

The components of the Dirac spinor Ψ are the $N \times N$ *complex* matrices $u_\pm$ and $v_\pm$, which are related to the left and right-moving spinor fields according to

$$\Psi_R = \frac{1}{2^{\frac{1}{4}}} \begin{pmatrix} u_+ \\ 0 \\ 0 \\ u_- \end{pmatrix} \quad \Psi_L = \frac{1}{2^{\frac{1}{4}}} \begin{pmatrix} 0 \\ v_+ \\ v_- \\ 0 \end{pmatrix} \tag{5}$$

Adopting the light-cone gauge $A_- = 0$ allows one to explicitly rewrite the left-moving fermion fields $v_\pm$ in terms of the right-moving fields $u_\pm$ and boson fields $\phi_\pm$, by virtue of equation (3). We may therefore eliminate $v_\pm$ dependence from the field theory. Moreover, Gauss' Law

$$\partial_-^2 A_+ = g\left(\mathrm{i}[\phi_+, \partial_-\phi_-] + \mathrm{i}[\phi_-, \partial_-\phi_+] + \{u_+, u_+^\dagger\} + \{u_-, u_-^\dagger\}\right) \tag{6}$$

permits one to remove any explicit dependence on A_+, and so the remaining *physical* degrees of freedom of the field theory are represented by the helicity $\pm\frac{1}{2}$ fermions $u_\pm$, and the helicity ± 1 bosons $\phi_\pm$. There are no ghosts in the quantization scheme adopted here. In the light-cone frame the Poincaré generators P^- and P^+ for the reduced $1+1$ dimensional field theory are given by

$$P^+ = \int_{-\infty}^{\infty} dx^- \mathrm{Tr}\left[2\partial_-\phi_- \cdot \partial_-\phi_+ + \frac{\mathrm{i}}{2}\sum_h \left(u_h^\dagger \cdot \partial_- u_h - \partial_- u_h^\dagger \cdot u_h\right)\right] \tag{7}$$

$$P^- = \int_{-\infty}^{\infty} dx^- \mathrm{Tr}\left[m_b^2 \phi_+\phi_- - \frac{g^2}{2} J^+ \frac{1}{\partial_-^2} J^+ + \frac{tg^2}{2}\left[\phi_+, \phi_-\right]^2 + \sum_h F_h^\dagger \frac{1}{\mathrm{i}\partial_-} F_h^\dagger\right] \tag{8}$$

where the sum $\sum_h$ is over $h = \pm$ helicity labels, and

$$J^+ = \mathrm{i}[\phi_+, \partial_-\phi_-] + \mathrm{i}[\phi_-, \partial_-\phi_+] + \{u_+, u_+^\dagger\} + \{u_-, u_-^\dagger\} \tag{9}$$

$$F_\pm = \mp sg\left[\phi_\pm\, , u_\mp\right] + \frac{m}{\sqrt{2}} u_\pm \tag{10}$$

We have generalized the couplings by introducing the variables t and s, which do not spoil the $1+1$ dimensional gauge invariance of the reduced theory;

the variable t will determine the strength of the quartic-like interactions, and the variable s will determine the strength of the Yukawa interactions between the fermion and boson fields, and appears explicitly in equation (10). The dimensional reduction of the original $3+1$ dimensional theory yields the canonical values $s = t = 1$.

Renormalizability of the reduced theory also requires the addition of a bare coupling m_b, which leaves the $1+1$ dimensional gauge invariance intact. In all calculations, the renormalized boson mass $\tilde{m}_b$ will be set to zero.

Canonical quantization of the field theory is performed by decomposing the boson and fermion fields into Fourier expansions at fixed light-cone time $x^+ = 0$:

$$u_\pm = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, b_\pm(k) e^{-\mathrm{i}kx^-} \quad \text{and} \quad \phi_\pm = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2|k|}}\, a_\pm(k) e^{-\mathrm{i}kx^-} \tag{11}$$

where $b_\pm = b_\pm^a \tau^a$ etc. We also define

$$b_\pm(-k) = d_\mp^\dagger(k), \quad a_\pm(-k) = a_\mp^\dagger(k), \tag{12}$$

where $d_\pm$ correspond to antifermions. Note that $(b_\pm^\dagger)_{ij}$ should be distinguished from $b_{\pm ij}^\dagger$, since in the former the quantum conjugate operator $\dagger$ acts on (color) indices, while it does not in the latter. The latter formalism is sometimes customary in the study of matrix models. The precise connection between the usual gauge theory and matrix theory formalism may be stated as follows:

$$b_{\pm ji}^\dagger = b_\pm^{a\dagger} \tau_{ji}^{a*} = b_\pm^{a\dagger} \tau_{ij}^a = (b_\pm^\dagger)_{ij}$$

The commutation and anti-commutation relations (in matrix formalism) for the boson and fermion fields take the following form in the large-N limit ($k, \tilde{k} > 0$; $h, h' = \pm$):

$$\left[a_{hij}(k), a_{h'kl}^\dagger(\tilde{k})\right] = \{b_{hij}(k), b_{h'kl}^\dagger(\tilde{k})\} = \{d_{hij}(k), d_{h'kl}^\dagger(\tilde{k})\} = \delta_{hh'}\delta_{jl}\delta_{ik}\delta(k-\tilde{k}), \tag{13}$$

where have used the relation $\tau^a_{ij}\tau^a_{kl} = \delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}$. All other (anti)commutators vanish.

The Fock space of physical states is generated by the color singlet states, which have a natural 'closed-string' interpretation. They are formed by a color trace of the fermion, antifermion and boson operators acting on the vacuum state $|0\rangle$. Multiple string states couple to the theory with strength $1/N$, and so may be ignored.

THE LIGHT CONE HAMILTONIAN

For the special case $\tilde{m}_b = m = t = s = 0$, the light-cone Hamiltonian is simply given by the current-current term $J^+ \frac{1}{\partial_-^2} J^+$ in equation (8). In momentum space, this Hamiltonian takes the form

$$
\begin{aligned}
P^-_{J^+.J^+} \quad = \quad & \frac{g^2}{2\pi}\int_{-\infty}^{\infty} dk_1 dk_2 dk_3 dk_4 \frac{\delta(k_1+k_2-k_3-k_4)}{(k_3-k_1)^2}\frac{\mathrm{Tr}}{2}\Bigg[\\
& \sum_{h,h'} : \{b^\dagger_h(k_1), b_h(k_3)\} :: \{b^\dagger_{h'}(k_2), b_{h'}(k_4)\} : \\
+ \quad & \frac{(k_1+k_3)(k_2+k_4)}{4\sqrt{|k_1||k_2||k_3||k_4|}} : [a^\dagger_+(k_1), a_+(k_3)] :: [a^\dagger_+(k_2), a_+(k_4)] : \\
+ \quad & \frac{(k_2+k_4)}{2\sqrt{|k_2||k_4|}} \sum_h : \{b^\dagger_h(k_1), b_h(k_3)\} :: [a^\dagger_+(k_2), a_+(k_4)] : \\
+ \quad & \frac{(k_3+k_1)}{2\sqrt{|k_1||k_3|}} \sum_{h'} : [a^\dagger_+(k_1), a_+(k_3)] :: \{b^\dagger_{h'}(k_2), b_{h'}(k_4)\} : \Bigg] \qquad (14)
\end{aligned}
$$

The explicit form of the Hamiltonian (14) in terms of the operators $b_\pm$, $d_\pm$ and $a_\pm$ is straightforward to calculate, but too long to be written down here. It should be stressed, however, that several $2 \to 2$ parton processes are suppressed by a factor $1/N$, and so are ignored in the large-N limit. No terms involving $1 \leftrightarrow 3$ parton interactions are suppressed in this limit, however.

One can show that this Hamiltonian conserves total helicity h, which is

an additive quantum number. Moreover, the number of fermions *minus* the number of antifermions is also conserved in each interaction, and so we have an additional quantum number $\mathcal{N}$. States with $\mathcal{N} = \textit{even}$ will be referred to as *boson* boundstates, while the quantum number $\mathcal{N} = \textit{odd}$ will refer to *fermion* boundstates. We will pay special attention to the cases $\mathcal{N} = 0$ and 3, since the associated states appear to be analogous to conventional mesons and baryons (respectively).

The instantaneous Coulomb interactions involving $2 \rightarrow 2$ parton interactions behave singularly when there is a zero exchange of momentum between identical 'in' and 'out' states. The same type of Coulomb singularity involving $2 \rightarrow 2$ boson-boson interactions appeared in a much simpler model [7], and can be shown to cancel a 'self-induced' mass term (or self-energy) obtained from normal ordering the Hamiltonian. The same prescription works in the model studied here. There are also finite residual terms left over after this cancellation is explicitly performed for the boson-boson and boson-fermion interactions, and they cannot be absorbed by a redefinition of existing coupling constants. These residual terms behave as momentum-dependent mass terms, and in some sense represent the flux-tube energy between adjacent partons in a color singlet state. For the boson-boson and boson-fermion interactions they are respectively

$$\frac{g^2 N}{2\pi} \cdot \frac{\pi}{4\sqrt{k_b k_{b'}}} \quad \text{and} \quad \frac{g^2 N}{2\pi} \frac{1}{k_f} \left(\sqrt{1 + \frac{k_f}{k_b}} - 1 \right) \tag{15}$$

where k_b, k_b' denote boson momenta, and k_f denotes a fermion momentum. These terms simply multiply the wavefunctions in the boundstate integral equations.

If we now include the contributions $F_h^\dagger \frac{1}{\mathrm{i}\partial_-} F_h$ in the light-cone Hamiltonian (8), then we will encounter another type of singularity for vanishing longitudinal momenta $k^+ = 0$. This singular behavior can be shown to cancel a (divergent) momentum-dependent mass term, which is obtained after normal

ordering the $F_h^\dagger \frac{1}{\mathrm{i}\partial_-} F_h$ interactions and performing an appropriate (infinite) renormalisation of the bare coupling m_b. This momentum-dependent mass term has the explicit form

$$\begin{aligned}
&\frac{s^2 g^2 N}{2\pi} \int_0^\infty dk_1 dk_2 \Bigg\{ \left(\frac{1}{k_2(k_1-k_2)} + \frac{1}{k_2(k_1+k_2)} \right) \sum_h a_h^\dagger(k_1) a_h(k_1) \\
&+ \quad \frac{1}{k_2(k_1-k_2)} \sum_h b_h^\dagger(k_1) b_h(k_1) + \frac{1}{k_2(k_1+k_2)} \sum_h d_h^\dagger(k_1) d_h(k_1) \Bigg\} \qquad (16)
\end{aligned}$$

The mechanism for cancellation here is different from the Coulombic case, since we will require specific endpoint relations relating different wavefunctions. Before outlining the general prescription for implementing this cancellation, we consider a simple rendering of the boundstate integral equations involving the $F_h^\dagger \frac{1}{\mathrm{i}\partial_-} F_h$ interactions. In particular, let us consider the helicity zero sector with $\mathcal{N} = 0$, and allow at most three partons. Then the boundstate integral equation governing the behavior of the wavefunction $f_{a_+a_-}(k_1, k_2)$ for the two-boson state $\frac{1}{N}\mathrm{Tr}[a_+^\dagger(k_1) a_-^\dagger(k_2)]|0\rangle$ takes the form

$$\begin{aligned}
M^2 f_{a_+a_-}(x_1, x_2) \;=\;& \frac{g^2 N}{\pi} \cdot \frac{\pi}{4\sqrt{x_1 x_2}} f_{a_+a_-}(x_1, x_2) \\
&+ \frac{s^2 g^2 N}{\pi} \sum_{i=1,2} \int_0^\infty dy \left(\frac{1}{y(x_i - y)} + \frac{1}{y(x_i + y)} \right) f_{a_+a_-}(x_1, x_2) \\
&- msg\sqrt{\frac{N}{2\pi}} \int_0^\infty d\alpha d\beta \, \delta(\alpha + \beta - x_1) \times \\
&\quad \frac{1}{\sqrt{x_1}} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) \left[f_{b_+d_+a_-}(\alpha, \beta, x_2) + f_{d_+b_+a_-}(\alpha, \beta, x_2) \right] \\
&\quad + \ldots \qquad (17)
\end{aligned}$$

where $M^2 = 2P^+P^-$, and $x_i = k_i/P^+$ are (boost invariant) longitudinal momentum fractions. Evidently, the integral (17) arising from $1 \to 2$ parton interactions behaves singularly for vanishing longitudinal momentum fraction $\alpha \to 0$, or $\beta \to 0$. However, these divergences are precisely canceled by the momentum-dependent mass terms, which represent the contribution (16).

To see this, we may consider the integral equation governing the wavefunction $f_{b_+d_+a_-}(k_1,k_2,k_3)$ for the three-parton state $\frac{1}{N^{3/2}}\mathrm{Tr}[b_+^\dagger(k_1)d_+^\dagger(k_2)a_-^\dagger(k_3)]|0\rangle$:

$$\begin{aligned} M^2 f_{b_+d_+a_-}(x_1,x_2,x_3) &= m^2\left(\frac{1}{x_1}+\frac{1}{x_2}\right) f_{b_+d_+a_-}(x_1,x_2,x_3) \\ &+ \frac{g^2N}{\pi}\sum_{i=1,2}\left[\frac{1}{x_i}\left(\sqrt{1+\frac{x_i}{x_3}}-1\right)\right] f_{b_+d_+a_-}(x_1,x_2,x_3) \\ &- msg\sqrt{\frac{N}{2\pi}}\frac{1}{\sqrt{x_1+x_2}}\left(\frac{1}{x_1}+\frac{1}{x_2}\right) f_{a_+a_-}(x_1+x_2,x_3) \\ &+ \ldots \end{aligned} \tag{18}$$

If we now multiply both sides of the above equation by x_i, and then let $x_i \to 0$ for $i = 1, 2$, we deduce the relations

$$f_{b_+d_+a_-}(0,x_2,x_3) = \frac{sg}{m}\sqrt{\frac{N}{2\pi}}\frac{f_{a_+a_-}(x_2,x_3)}{\sqrt{x_2}} \tag{19}$$

$$f_{b_+d_+a_-}(x_1,0,x_3) = \frac{sg}{m}\sqrt{\frac{N}{2\pi}}\frac{f_{a_+a_-}(x_1,x_3)}{\sqrt{x_1}} \tag{20}$$

It is now straightforward to show that the singular behavior of the integral (17) involving the wavefunction $f_{b_+d_+a_-}$ may be written in terms of a momentum-dependent mass term involving the wavefunction $f_{a_+a_-}$. Similar divergent contributions are obtained from the the wavefunctions $f_{d_+b_+a_-}$, $f_{a_+b_-d_-}$ and $f_{a_+d_-b_-}$, all of which may be re-expressed in terms of the wavefunction $f_{a_+a_-}$ by virtue of corresponding 'ladder relations'. The sum of these divergent contributions exactly cancels the self-energy contribution. An entirely analogous set of ladder relations were found for the case of fermions in the fundamental representation of $SU(N)$ [2].

For the general case where states are permitted to have more than three partons, the correct ladder relations are not immediately obvious from an analysis of the integral equations alone. Nevertheless, they may be readily

obtained from the constraint equation governing the left-moving fermion field Ψ_L. In particular, we have $\mathrm{i}\partial_- v_\mp = F_\pm$, and so vanishing fields at spatial infinity would imply

$$\int_{-\infty}^{\infty} dx^- F_\pm |\Psi\rangle = 0 \tag{21}$$

for color singlet states $|\Psi\rangle$. The analysis of this condition in momentum space is quite delicate, since it involves integrals of singular wavefunctions over spaces of measure zero [8]. Viewed in this way we see that the ladder relations are the continuum equivalent of zero mode constraint equations that have shown to lead to spontaneous symmetry breaking in discrete light-cone quantization [9].

EXACT SOLUTIONS

For the special case $s = m = \tilde{m}_b = 0$, the only surviving terms in the Hamiltonian (8) are the current-current interactions $J^+ \frac{1}{\partial_-^2} J^+$ and the ϕ^4 interaction. This theory has infinitely many massless boundstates, and the partons in these states are either fermions or antifermions. States with bosonic $a_\pm$ quanta are always massive. One also finds that the massless states are pure, in the sense that the number of partons is a fixed integer, and there is no mixing between sectors of different parton number. In particular, for each integer $n \geq 2$, one can always find a massless boundstate consisting of a superposition of only n-parton states. A striking feature is that the wavefunctions of these states are *constant*, and so these states are natural generalizations of the constant wavefunction solution appearing in t'Hooft's model [10].

We present an explicit example below of such a constant wavefunction solution involving a three fermion state with total helicity $+\frac{3}{2}$, which is perhaps the simplest case to study. Massless states with five or more partons appear to have more than one wavefunction which are non-zero and constant, and in general the wavefunctions are unequal. It would be interesting to classify

all states systematically, and we leave this to future work. One can, however, easily count the number of massless states. In particular, for $\mathcal{N} = 3$, $h = +\frac{3}{2}$ states, there is one three-parton state, 2 five-parton states, 14 seven-parton states and 106 nine-parton states that yield massless solutions.

Let us now consider the action of the light-cone Hamiltonian P^- on the three-parton state

$$\begin{aligned} |b_+b_+b_+\rangle &= \int_0^\infty dk_1 dk_2 dk_3 \; \delta(\sum_{i=1}^{3} k_i - P^+) f_{b_+b_+b_+}(k_1,k_2,k_3) \\ & \frac{1}{N^{3/2}} \mathrm{Tr}[b_+^\dagger(k_1) b_+^\dagger(k_2) b_+^\dagger(k_3)]|0\rangle \end{aligned} \tag{22}$$

The quantum number $\mathcal{N}$ is 3 in this case, and ensures that the state $P^-|b_+b_+b_+\rangle$ must have at least three partons. In fact, one can deduce the following:

$$\begin{aligned}
& P^- \mid b_+b_+b_+\rangle = \int_0^\infty dk_1 dk_2 dk_3 \; \delta(\sum_{i\,=1}^{3} k_i - P^+) \\
& -\frac{g^2 N}{2\pi} \int_0^\infty d\alpha d\beta \frac{\delta(\alpha+\beta-k_1-k_2)}{(\alpha-k_1)^2} \Big[f_{b_+b_+b_+}(\alpha,\beta,k_3) - f_{b_+b_+b_+}(k_1,k_2,k_3) \Big] \\
& \frac{1}{N^{3/2}} \mathrm{Tr} \Big[b_+^\dagger(\alpha) b_+^\dagger(\beta) b_+^\dagger(k_3) \Big] \mid 0\rangle \\
& \\
& +\frac{g^2 N}{2\pi} \int_0^\infty d\alpha d\beta d\gamma \sum_h \frac{\delta(\alpha+\beta+\gamma-k_1)}{(\alpha+\beta)^2} f_{b_+b_+b_+}(\alpha+\beta+\gamma,k_2,k_3) \frac{1}{N^{5/2}} \mathrm{Tr} \\
& \Big[\{b_h^\dagger(\alpha), d_{-h}^\dagger(\beta)\} b_+^\dagger(\gamma) b_+^\dagger(k_2) b_+^\dagger(k_3) - \{b_h^\dagger(\alpha), d_{-h}^\dagger(\beta)\} b_+^\dagger(k_2) b_+^\dagger(k_3) b_+^\dagger(\gamma) \Big] \mid 0\rangle \\
& \\
& +\frac{g^2 N}{4\pi} \int_0^\infty d\alpha d\beta d\gamma \sum_h \frac{\delta(\alpha+\beta+\gamma-k_1)}{\sqrt{\alpha\beta}(\alpha+\beta)^2} f_{b_+b_+b_+}(\alpha+\beta+\gamma,k_2,k_3) \frac{1}{N^{5/2}} \mathrm{Tr} \\
& \Big[[a_h^\dagger(\alpha), a_{-h}^\dagger(\beta)] b_+^\dagger(\gamma) b_+^\dagger(k_2) b_+^\dagger(k_3) - [a_h^\dagger(\alpha), a_{-h}^\dagger(\beta)] b_+^\dagger(k_2) b_+^\dagger(k_3) b_+^\dagger(\gamma) \Big] \mid 0\rangle \\
& + \text{cyclic permutations} \Big\}
\end{aligned} \tag{23}$$

The five-parton states above correspond to virtual fermion-antifermion and boson-boson pair creation. The expression (23) vanishes if the wavefunction $f_{b_+b_+b_+}$ is constant.

CONCLUTIONS

We have presented a non-perturbative Hamiltonian formulation of a class of $1+1$ dimensional matrix field theories, which may be derived from a classical dimensional reduction of QCD_{3+1} coupled to Dirac adjoint fermions. We choose to adopt the light-cone gauge $A_- = 0$, and are able to solve numerically the boundstate integral equations in the large-N limit. Different states may be classified according to total helicity h, and the quantum number $\mathcal{N}$, which defines the number of fermions minus the number of antifermions in a state.

For a special choice of couplings that eliminates all interactions except those involving the longitudinal current J^+ and the ϕ^4 interactions we find an infinite number of pure massless states of arbitrary length. The wavefunctions of these states are always constant, and may be solved for exactly and an example was explicitly given. In general, a massless solution involves several (possibly different) constant wavefunctions. The massless solutions observed in studies of $1+1$ dimensional supersymmetric field theories [13] are not analogous to the constant wavefunction solutions found here.

When one includes the Yukawa interactions, singularities at vanishing longitudinal momenta arise, and we show in a simple case how these are canceled by the boson and fermion self-energies. This cancellation relies on the derivation of certain 'ladder relations', which relate different wavefunctions at vanishing longitudinal momenta. These relations become singular for vanishing fermion mass m, and so in the context of the numerical techniques employed here, one is prevented from studying the limit $m \to 0$. Analytical techniques which are currently under investigation are expected to be

relevant in this limiting case [8].

A particularly important property of these models is that virtual pair creation and annihilation of bosons and fermions is not suppressed in the large-N limit, and so our results go beyond the valence quark (or quenched) approximation. This provides the scope for strictly field-theoretic investigations of the internal structure of boundstates where 'sea-quarks' and small-x gluons are expected to contribute significantly to the overall polarization of a boundstate.

The techniques employed here are not specific to the choice of field theory, and are expected to have a wide range of applicability, particularly in the light-cone Hamiltonian formulation of supersymmetric field theories.

ACKNOWLEDGMENTS

This work was done in collaboration with Francesco Antonuccio. The work was supported in part by a grant from the US Department of Energy. Travel support was provided in part by a NATO collaborative grant.

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